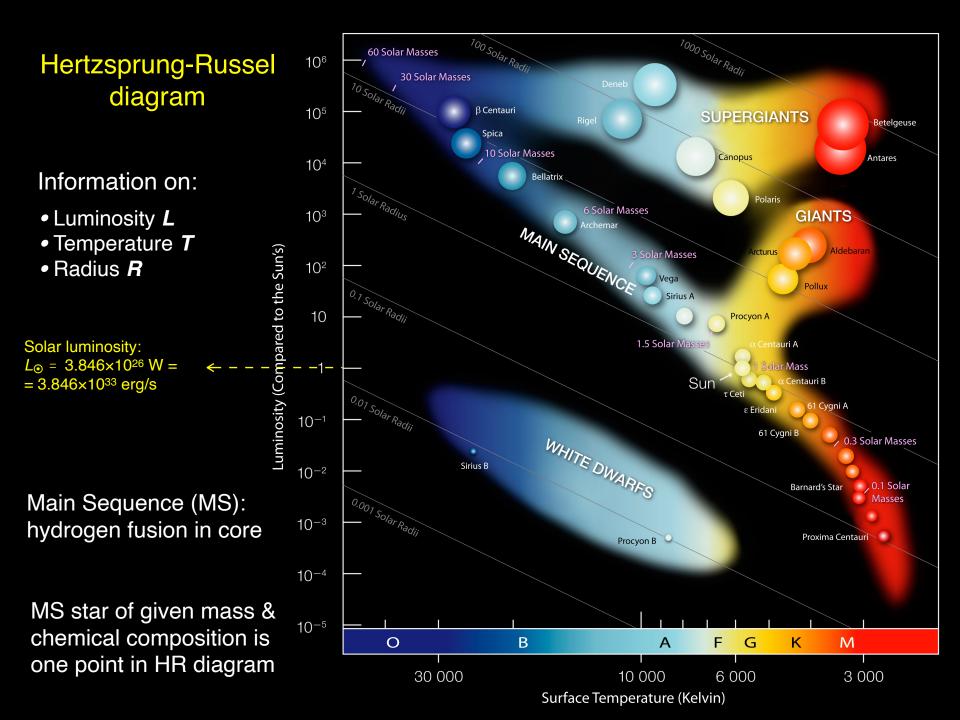
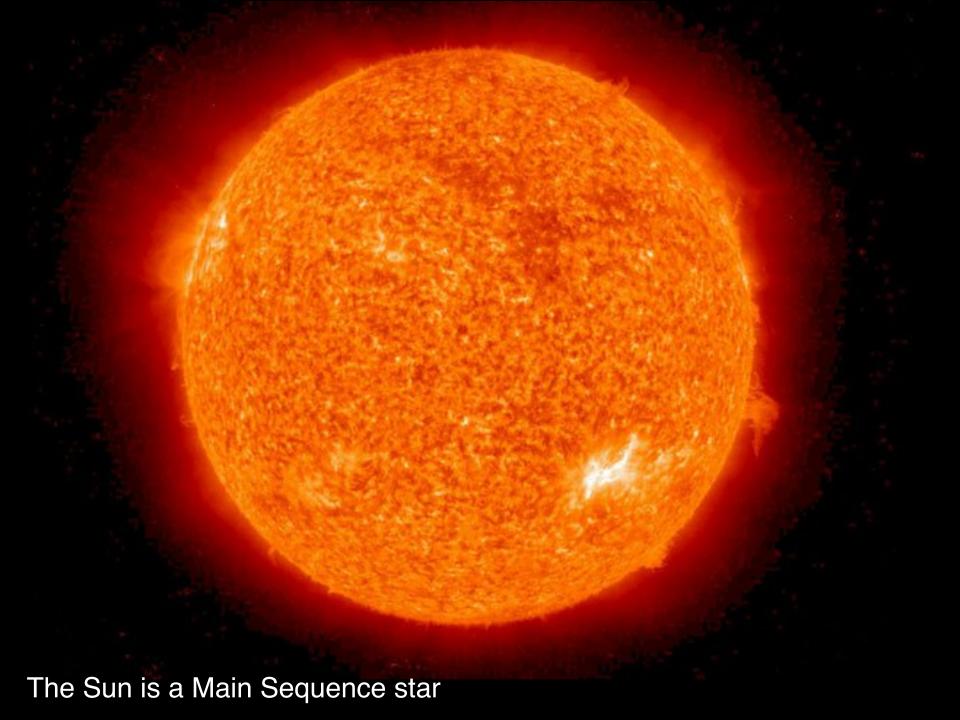
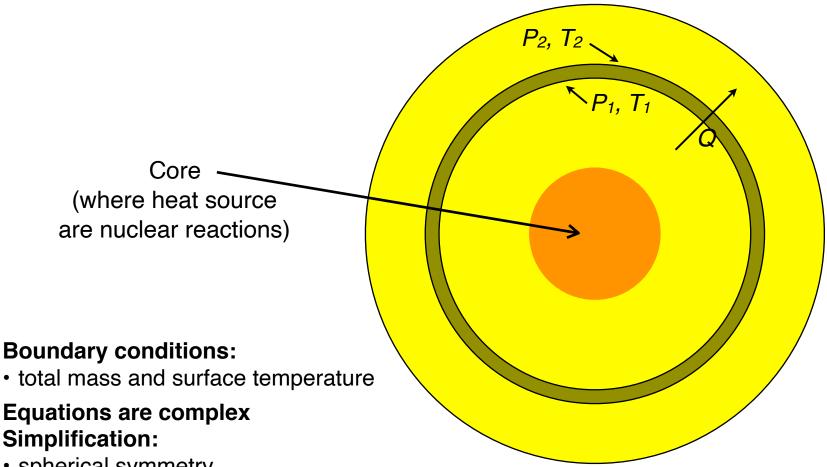
Stars in the Main Sequence of the Hertzsprung-Russel diagram





Equation of stellar structure gives everywhere *P*, *T*, and *n*

Hydrostatic equilibrium: going to center, gravity balanced by pressure (gas + radiation)



spherical symmetry

ideal gas

thin spherical shell

• $P_1 > P_2 \to T_1 > T_2$

energy flow is generated at interface (from center)

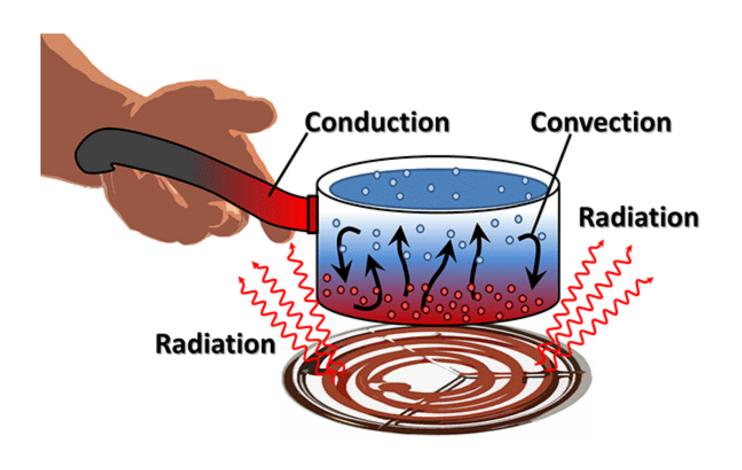
P: pressure

T: temperature

n : particle density

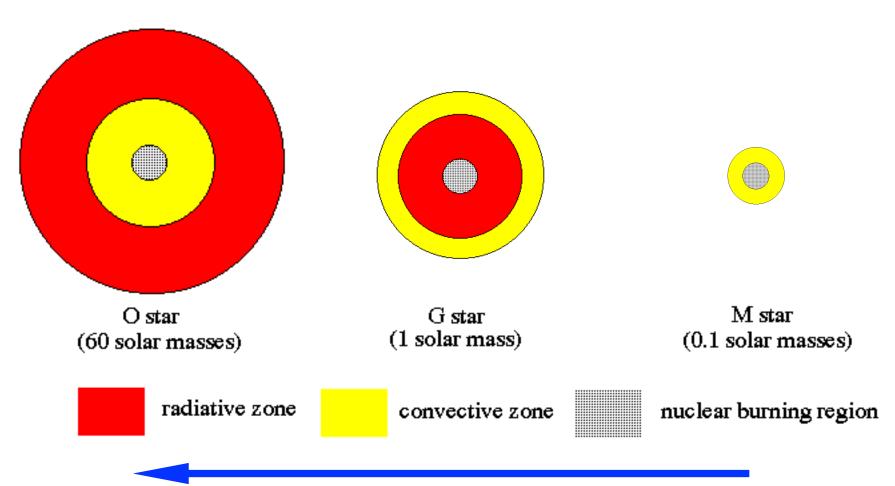
Q: energy flow (heat)

Three main ways for heat transfer



Different stellar structure for different star masses

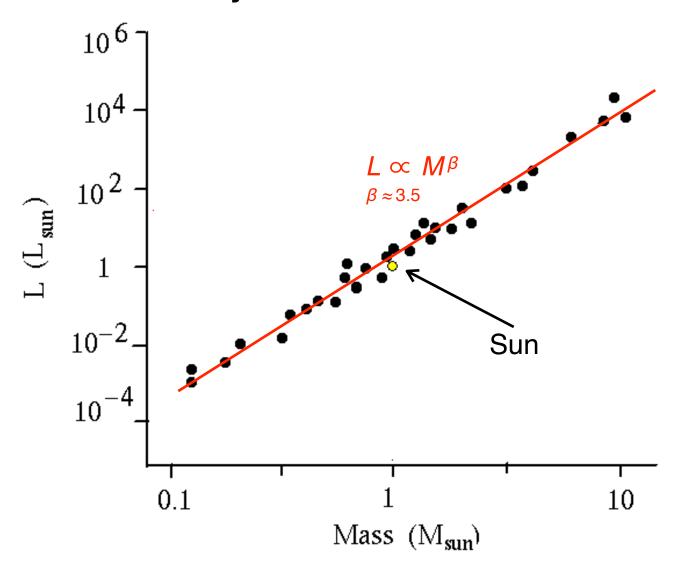
Internal Structure for Main Sequence Stars



Stellar mass

Sun dominated by radiation, dwarf stars dominated by convection

Mass-luminosity relation for stars in Main Sequence



Most commonly slope used to estimate stellar lifetimes: $\beta = 3.5$ In more realistic approach: $3 < \beta < 4$

Key Properties of Main Sequence Stars

Mass/M _⊙	Luminosity/L _⊙	Effective Temperature (K)	Radius/ <i>R</i> ⊙	Main Sequence lifespan (yrs)
0.10	3×10-3	2900	0.16	2×10 ¹²
0.50	0.03	3800	0.6	2×10 ¹¹
0.75	0.3	5000	0.8	3×10 ¹⁰
1.0	1	6000	1	1×10 ¹⁰
1.5	5	7000	1.4	2×109
3	60	11000	2.5	2×108
5	600	17000	3.8	7×10 ⁷
10	10,000	22000	5.6	2×10 ⁷
15	17,000	28000	6.8	1×10 ⁷
25	80,000	35000	8.7	7×106

15

3.4×106

The Sun

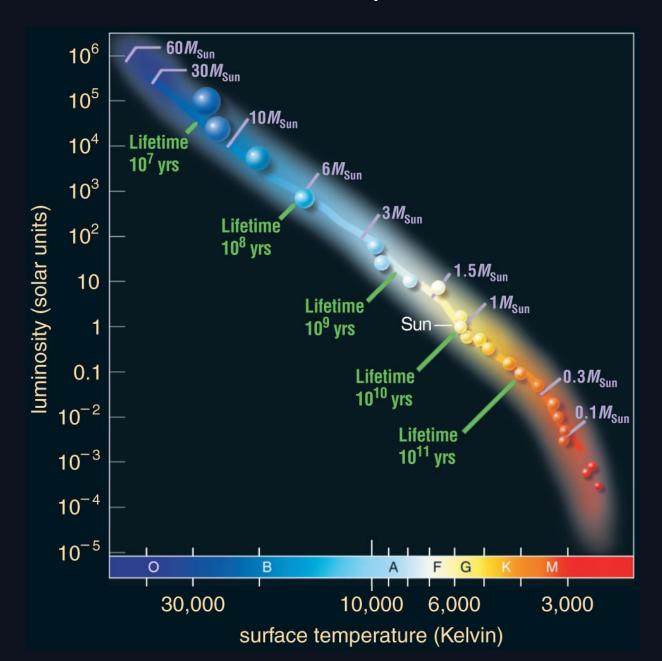
60

790,000

Effective temperature: surface temperature of a star assuming black body emissivity

44500

Lifetime of stars in Main Sequence with different mass



Nuclear reactions in stellar cores

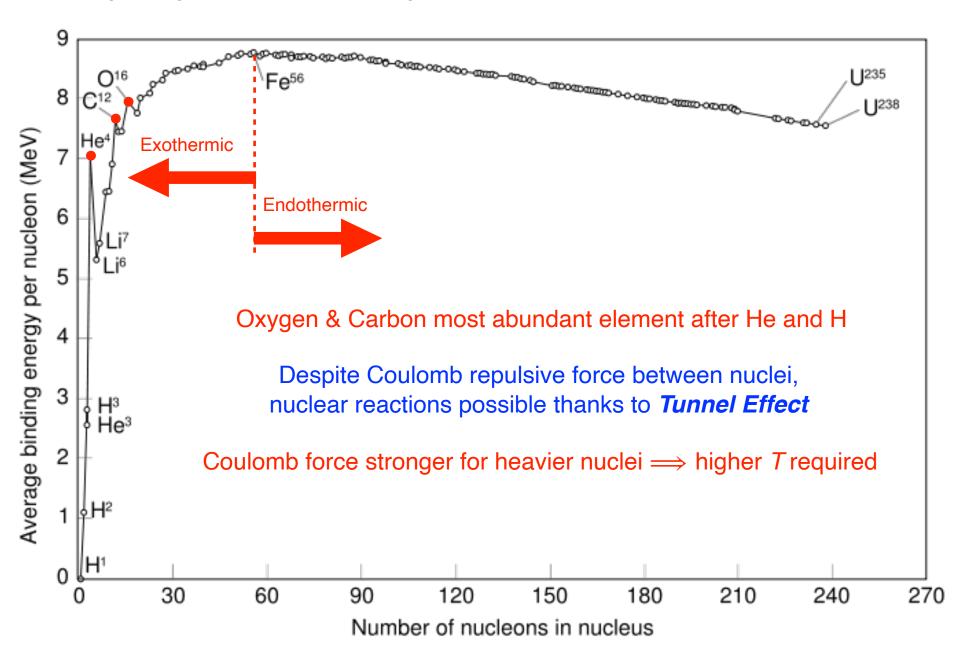
Chemical composition of stars very similar to Sun's

Element abundances in the Sun

Element	#particles	Mass(X) / Mass(total)
Н	92.1%	0.74
He	7.8%	0.25
Z	0.1%	0.014

Z: all chemical elements heavier than helium

Binding energy (or rest frame energy) per nucleon, as a function of mass number



Hydrogen fusion in core of Main Sequence stars

Two main ways: proton-proton (pp) chains & CNO Cycle

pp I:
$$4p \rightarrow {}^{4}_{2}He + 2e^{+} + 2\nu_{e} + 2\gamma$$

pp II: $4p + e^{-} \rightarrow {}^{4}_{2}He + e^{+} + 2\nu_{e} + 2\gamma$
pp III: $4p \rightarrow {}^{4}_{2}He + 2e^{+} + 2\nu_{e} + 3\gamma$
CNO: $4p \rightarrow {}^{4}_{2}He + 2e^{+} + 2\nu_{e} + 2\gamma$

1 He atom formed from 4 protons

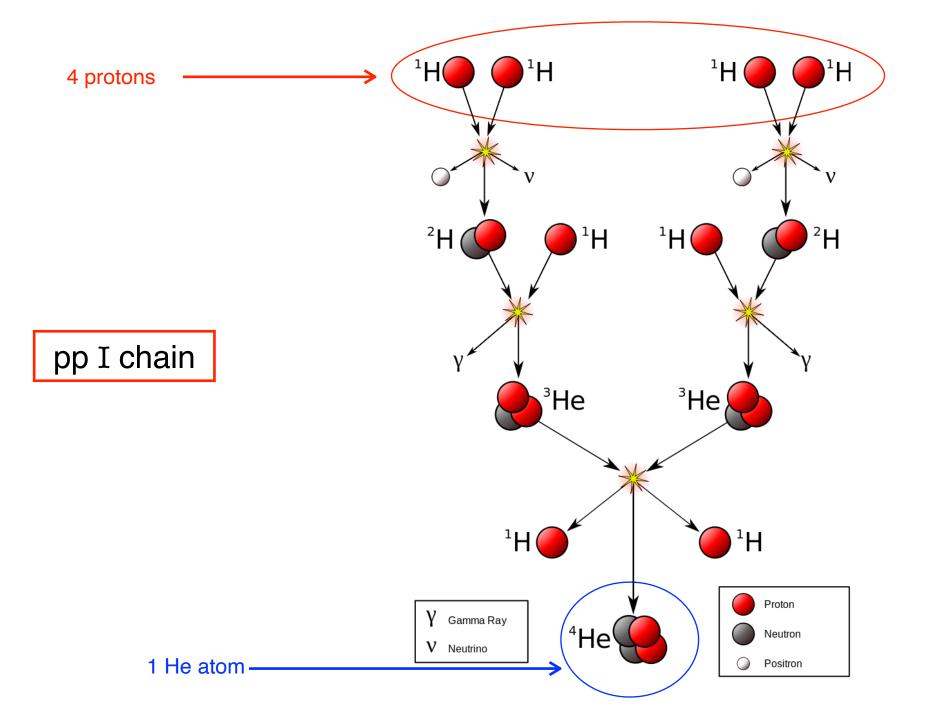
ν_e: electron neutrino

γ: gamma-ray photon

p: proton

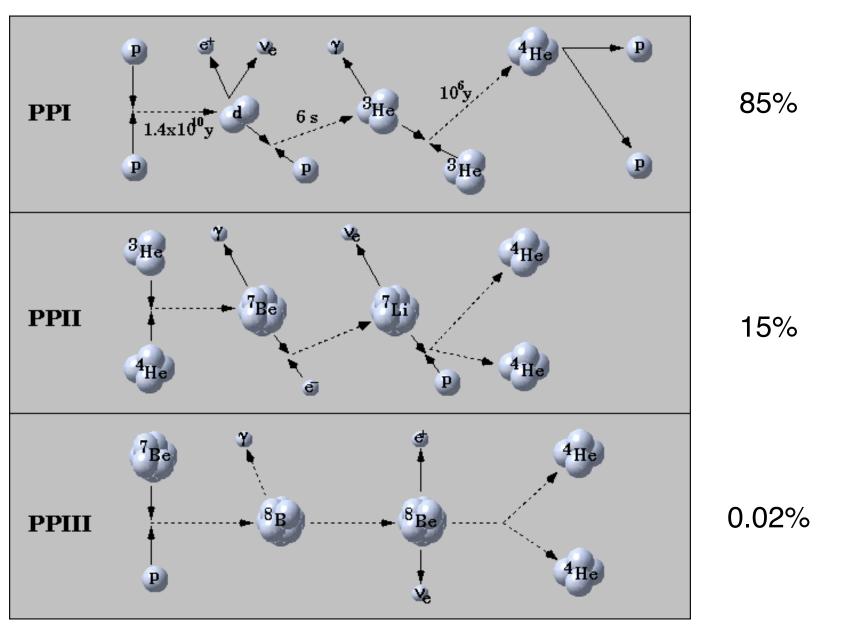
CNO: carbon nitrogen oxygen

Total energy released for one pp chain: $E = 26.73 \text{ MeV} = 4.28 \times 10^{-12} \text{ J}$

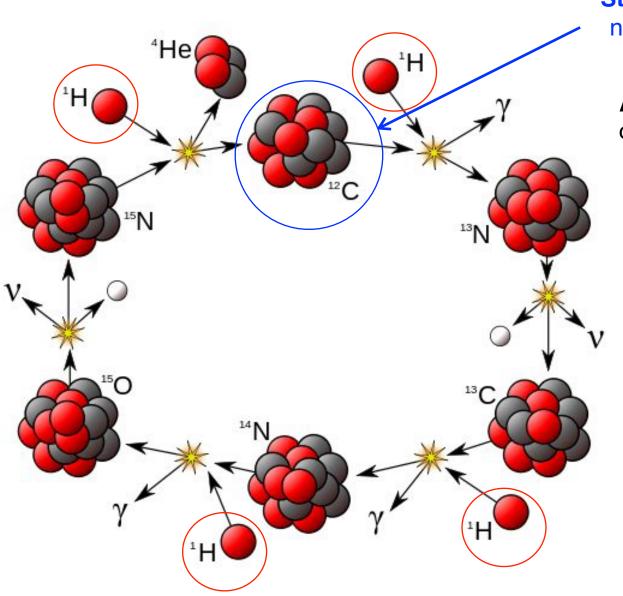


The 3 pp chains

For the Sun:

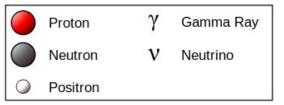


CNO cycle

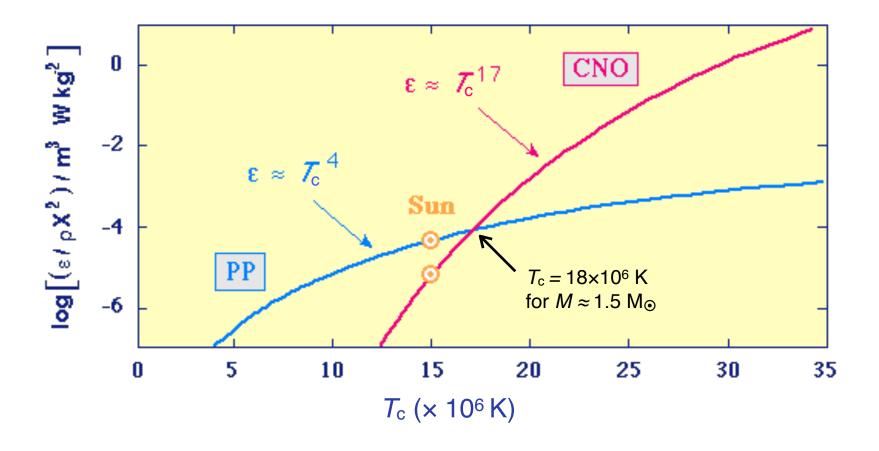


Start: ¹²C acts as nuclear catalyst

Atoms used as catalysts: carbon, nitrogen, oxygen

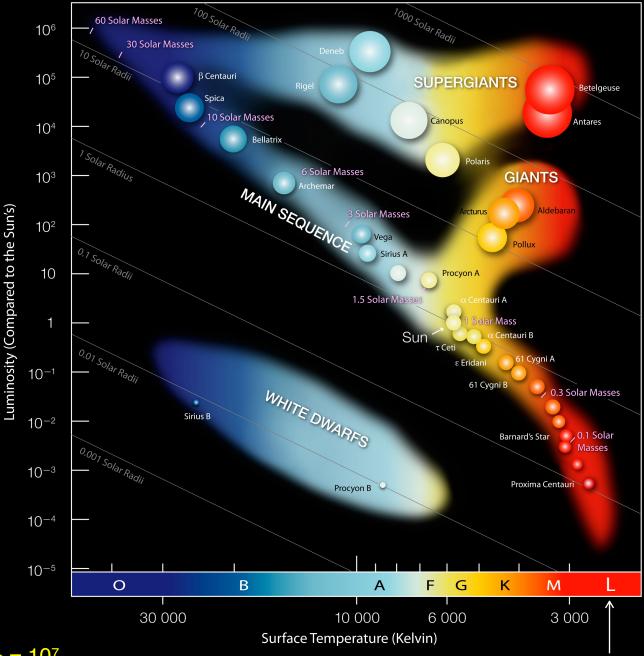


Rate of **energy release** as a function of core temperature T_c for **pp chains** and **CNO cycle**



Every second, Sun converts 4×10⁹ kg of H to energy (radiated to space) **Hydrogen** in Sun's core will be **exhausted** in another **5 billion years**

Hertzsprung-Russel diagram

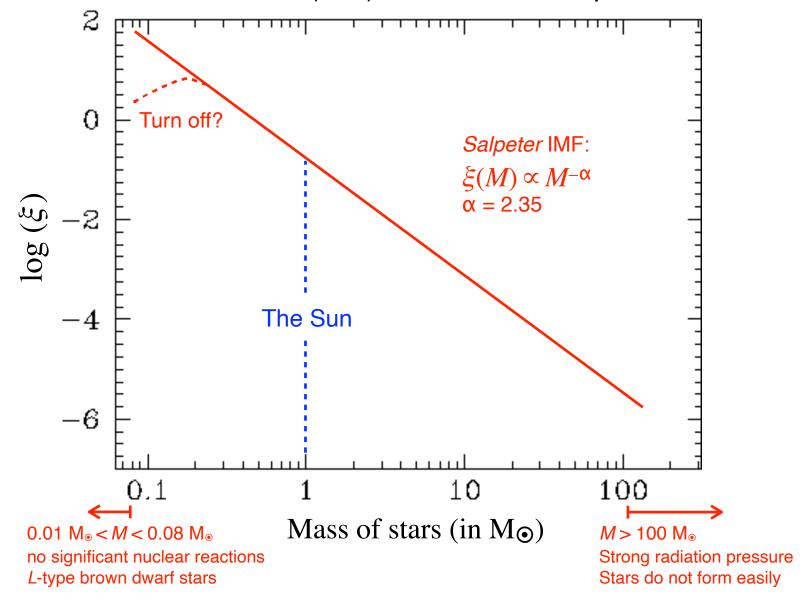


Nuclear reactions strong function of T_c (then mass)

Thus: $M_1/M_2 = 100 \Longrightarrow L_1/L_2 = 10^7$

Brown dwarf stars

Stellar initial mass function (IMF): number of stars per mass interval



L-type stars are 2× more numerous than all other stars but count for 15% of stellar mass in MW

Smallest stars (L-type, M < 0.08 M $_{\odot}$ = 80 M $_{\rm J}$) nuclear reactions are not very significant \Longrightarrow brown dwarf stars visible in IR (T < 2600 K)

HST image of *Trapezium Cluster* in Orion nebula

Cool stars are bright in the IR



Optical Infrared

> Mass lower limit 0.013 M_☉ (14× Jupiter mass) minimum required to start nuclear fusion of deuterium

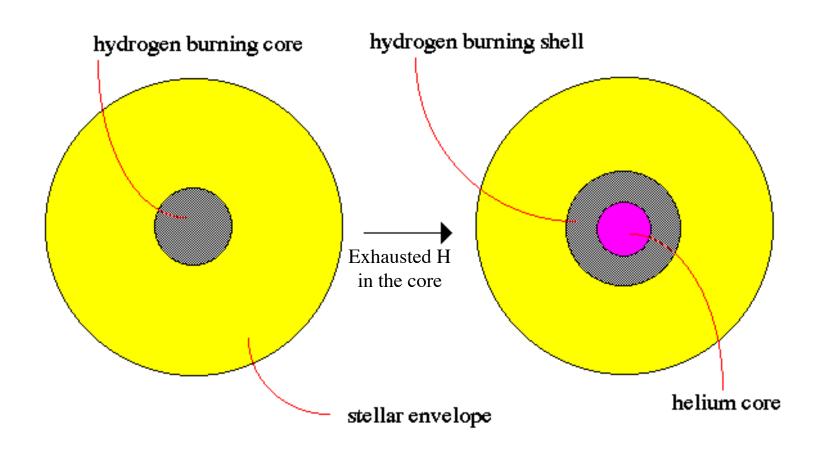
Star life beyond the Main Sequence (hydrogen exhaustion in core)

Star leaves MS when hydrogen fusion ends in core

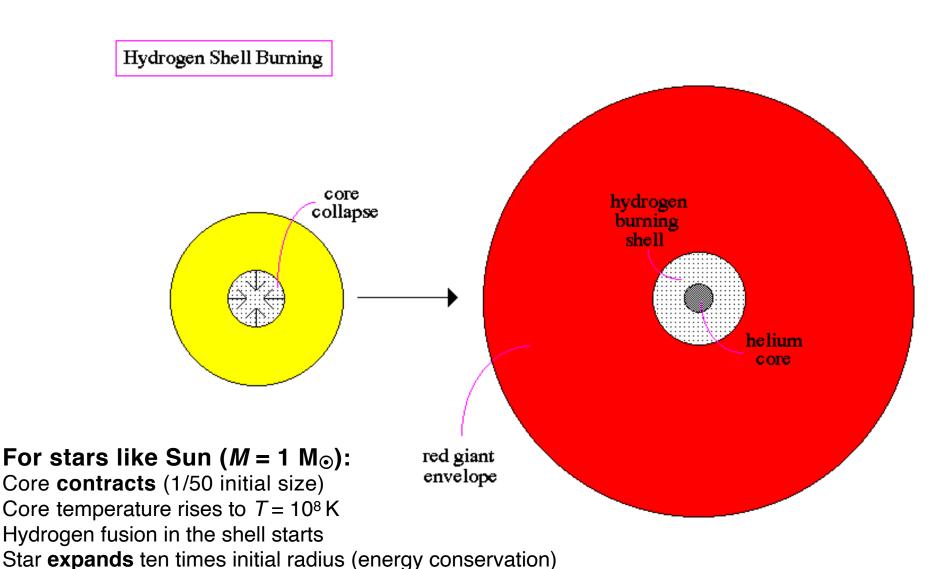
For star mass $M < 0.5 \, \mathrm{M}_{\odot}$, no helium burning in the core **Degeneracy pressure** of electrons (Pauli exclusion principle for fermions) stops contraction Degeneracy pressure does not depend on temperature but on **density only**

For masses 1 M $_{\odot}$ < M < 2 M $_{\odot}$ \rightarrow hydrogen burning in the shell

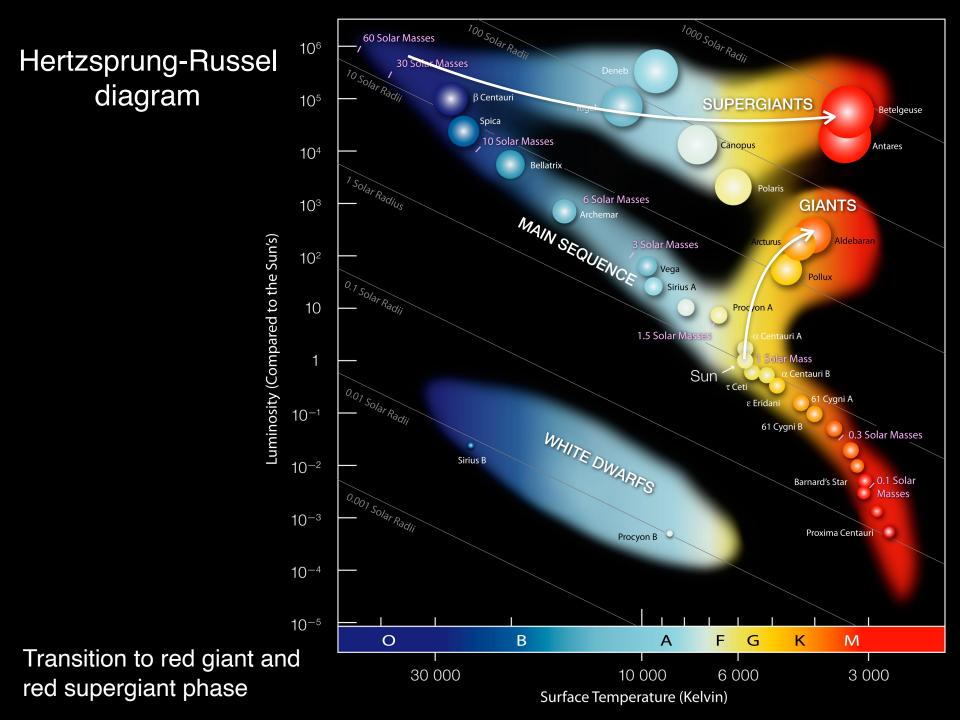
Core Exhaustion



After MS phase: T drops \rightarrow core collapse \rightarrow gravitation energy converted into heat \rightarrow H burning in shell around core \rightarrow expanding envelope \rightarrow Red Giant



Surface temperature drops: T = 3500 K (red giant star)



After red giant, for $M > 0.5 \text{ M}_{\odot} \rightarrow T = 10^8 \text{ K}$ in core \rightarrow **He fusion**

He fusion (triple-alpha process) to produce carbon

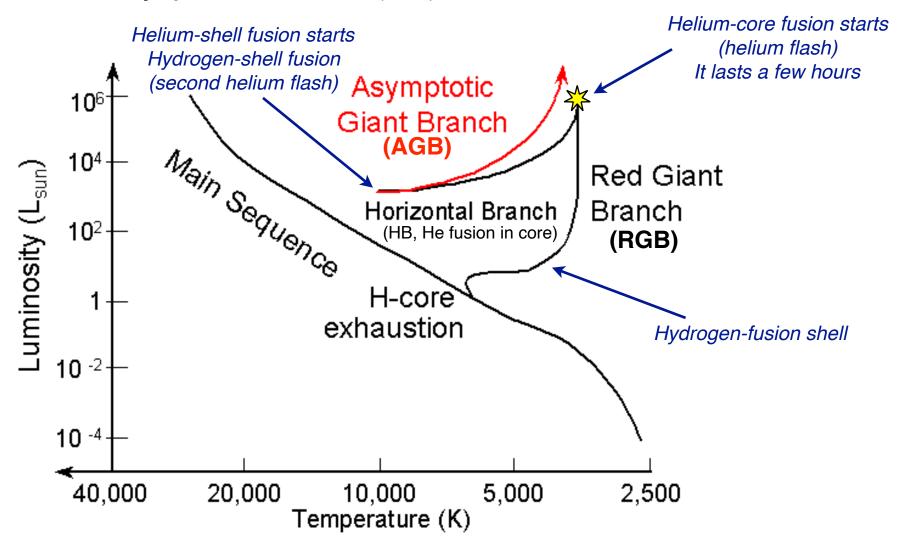
3α process in He core:

$${}^{4}He + {}^{4}He \rightleftharpoons {}^{8}Be$$
 ${}^{4}He + {}^{8}Be \rightleftharpoons {}^{12}C^{*} \rightarrow {}^{12}C + 2\gamma$

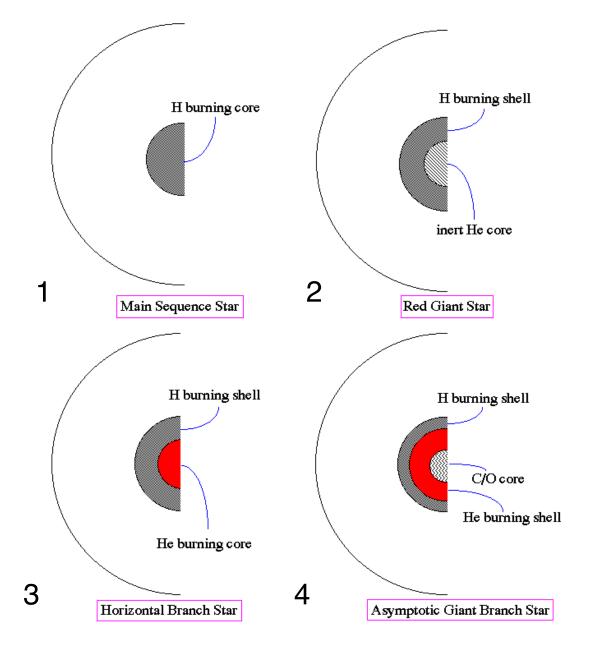
- E released 1.17 \times 10⁻¹² J per C nucleus (or 3.9 \times 10⁻¹³ J per He nucleus)
- This is 10% of energy with H fusion (10 times faster)
- Energy released very temperature sensitive, $E \propto T^{40}$, necessary to have Be + He before Be goes back to He

Evolution of stars in Hertzsprung-Russel diagram for medium-size star ($M < 2 \text{ M}_{\odot} \rightarrow \text{He flash in core}$)

- Then He fusion in core (Horizontal Branch)
- At end of HB, core of inert carbon and oxygen
- Electrons and nuclei in core again become degenerate, star expands and cools
 - → **Asymptotic Giant Branch** (AGB)



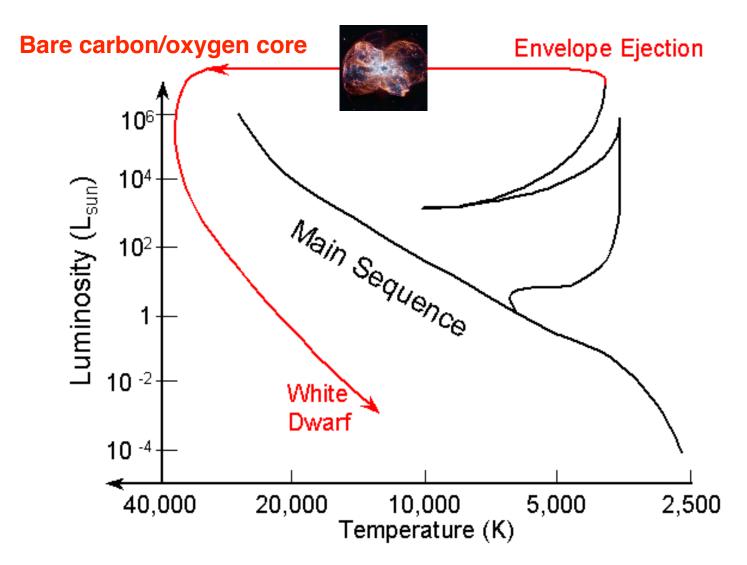
Evolution of a stellar core for medium-size star: summary



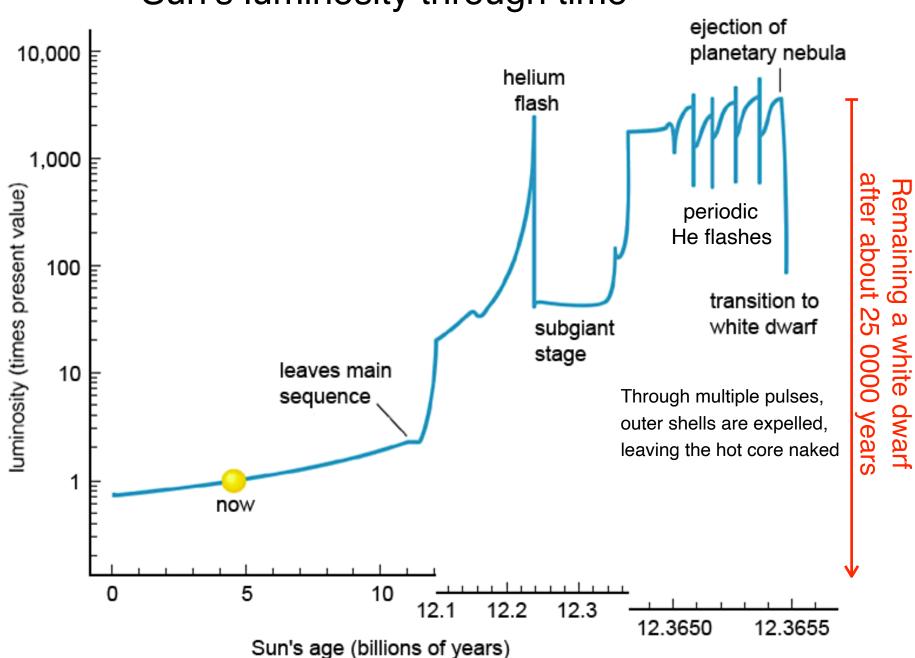
During He fusion, small changes in $T \rightarrow$ large changes in energy release ($E \propto T^{40}$)

Huge thermal pulses destabilize outer envelope: Planetary Nebula phase

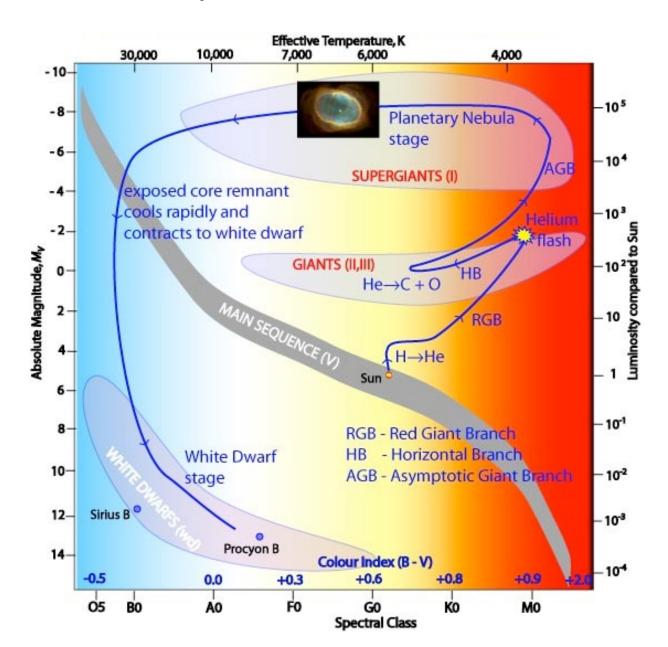
Planetary nebula: large tenuous gas shells expanding at $v \sim$ a few km/s enough against gravity, gas dissipates in outer space in \sim 20,000 yr



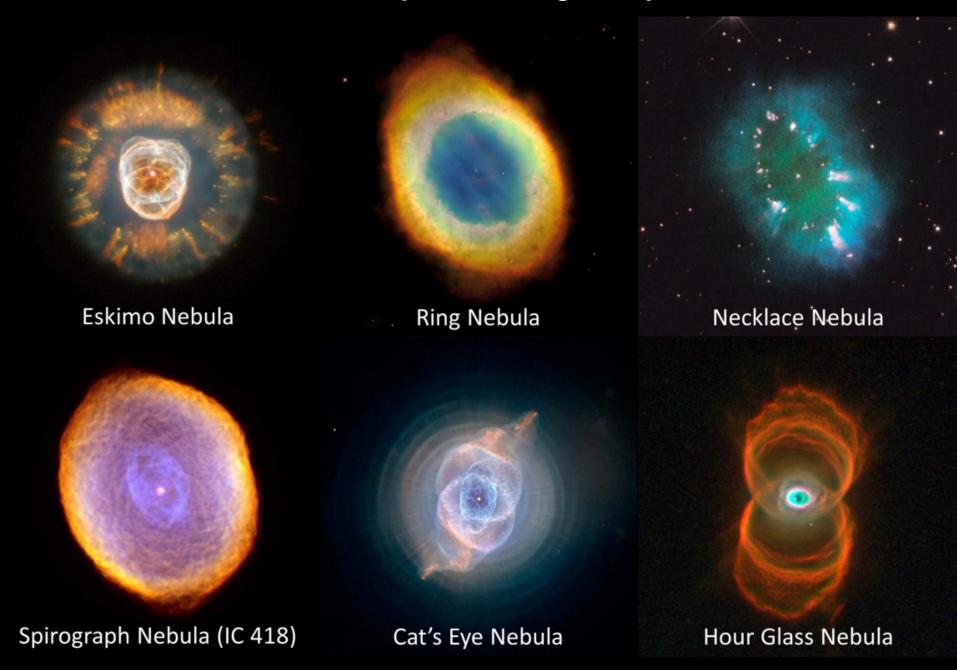
Sun's luminosity through time



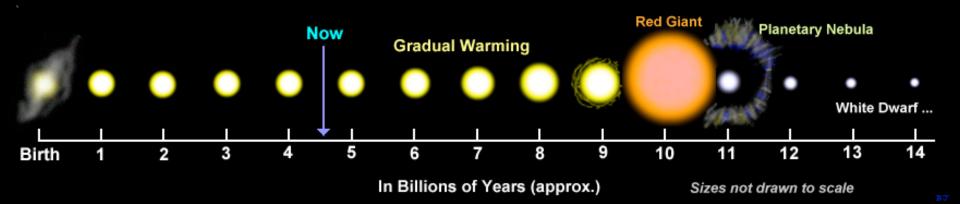
Sun's post-MS time evolution



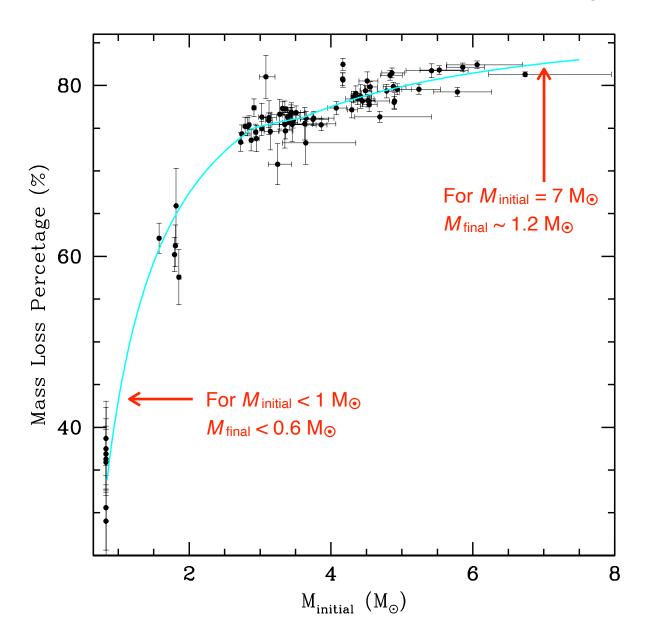
Planetary nebula gallery



Life cycle of the Sun



Total mass loss before white dwarf remnant as a percentage of initial mass



Stellar wind & mass loss in massive stars

Betelgeuse

Mass: $M = 15 - 20 M_{☉}$

Surface temperature: *T* ~ 3600 K

Radius: R ~ 1000 R_☉

Stage before massive explosion (supernova)

Dust forms in atmosphere of red giant stars

Thanks to: low gravity high gas density low temperature

Size of Star

Size of Earth's Orbit

Size of Jupiter's Orbit

Mass loss in massive stars due to radiation pressure

Mass loss rate:

 $M = 100 \text{ M}_{\odot}$ star: up to 1/2 mass before end of MS

O stars: 10⁻⁶ M_⊙ / yr The Sun: 10⁻¹⁴ M_☉ / yr

Wolf-Rayet star WR124

Distance: 3.35 kpc Mass: *M* = 9 M_☉

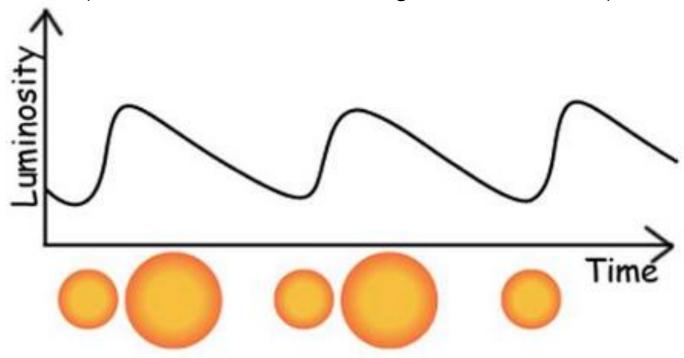
Temperature: T = 35,900 K

Age: 8.6 Myr



Variable stars: Cepheids

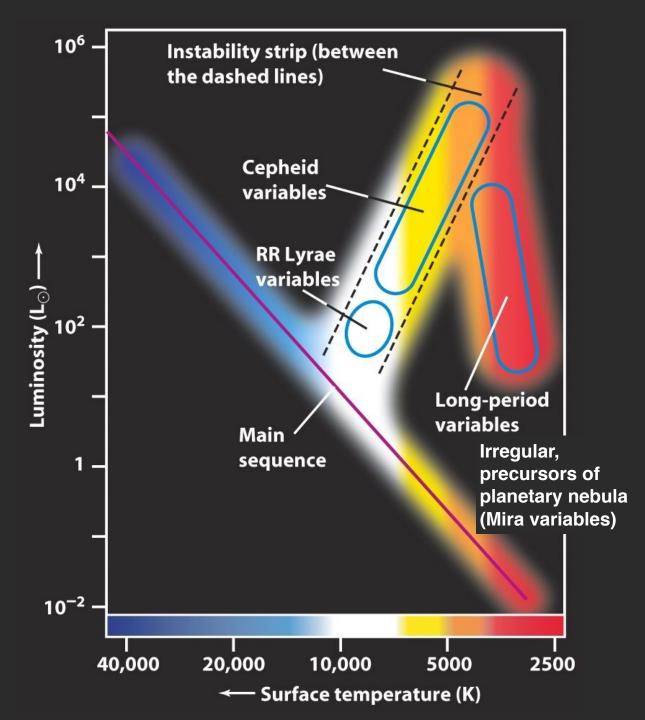
(masses are 4–20 times higher than the Sun)



- Stars in AGB (He-shell burning) developing **thermal pulses** (*R* varies by more than 10%)
- At distance from center where T = 40,000 K → He⁺ → He²⁺ → free electrons interacting with radiation (gas less transparent) → radiation trapped inside → P goes up, layers inside expands, density drops → layer gets transparent again
- Regular cycle repeats (pulsation)
- During pulsation, large mass loss (10⁻⁶ M_☉ / yr), dust grains and molecules formation (relatively high density and low temperature in outflow) → circumstellar shell formation
- · Pulsation is short in star's lifetime

Cepheid variable star RS Puppis Apparent magnitude: $m_V = 6.5-7.6$ Mass: *M* = 9.2 M_☉ Period of pulsation: 41.5 days Radius variation: $R = (164-208) R_{\odot}$ Distance: 1.7 kpc





Evolution of massive stars after MS

When $T = 3 \times 10^8$ K in core in red giant stars (M < 8 M $_{\odot}$), carbon fusion starts

3α (**triple alpha**) process in He core for **carbon production**:

$${}^{4}He + {}^{4}He \rightleftharpoons {}^{8}Be$$
 ${}^{4}He + {}^{8}Be \rightleftharpoons {}^{12}C^{*} \rightarrow {}^{12}C + 2\gamma$

Then carbon fusion for **oxygen production**:

$$^{4}He + ^{12}C \rightarrow ^{16}O + \gamma$$

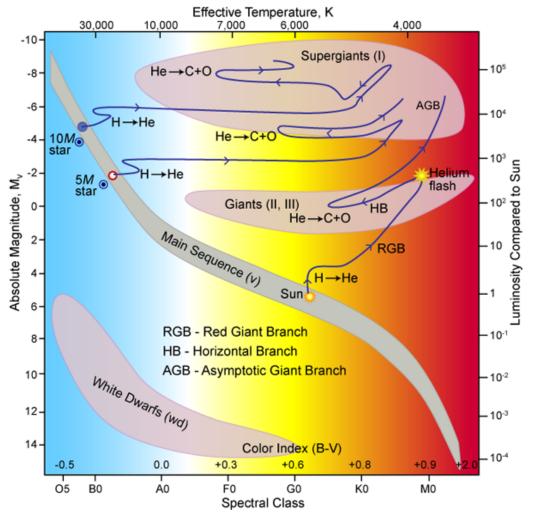
These two processes main source of C and O in the universe For $M < 8 \text{ M}_{\odot}$, other reactions not triggered (core left with O & C)

Evolution of massive stars $(M > 5 \text{ M}_{\odot})$ after MS

Color changes \rightarrow *T* drops to 4000 K (red)

Luminosity doesn't change much $\rightarrow R$ gets very large (*Betelgeuse*: $R = 1000 \text{ R}_{\odot}$ for $M = 20 \text{ M}_{\odot}$)

Due to mass loss (winds or in binary systems), color can change from blue to red and back



For M > 5 M_{\odot}, radiation pressure main support to star

Element production in massive stars (mass limits not precisely known!)

• For $M > 5 M_{\odot}$:

When He exhausted in the core, T goes up to 3×10^8 K, then oxygen production:

$$^{12}_{6}\text{C} + ^{4}_{2}\text{He} \rightarrow ^{16}_{8}\text{O} + \gamma$$

This, 3α and H shell burning main sources of energy

• For M > 8 M_{\odot}, carbon fusion in core (elements with mass number $A \sim 20$ formed):

 $T = 10^9$ K, **photo-disintegration** of nuclei, reverse reaction. Moreover, **neon burning** to give Mg:

$$^{20}_{10}\text{Ne} + \gamma \rightarrow ^{16}_{8}\text{O} + ^{4}_{2}\text{He} \rightarrow ^{20}_{10}\text{Ne} + ^{4}_{2}\text{He} \rightarrow ^{24}_{12}\text{Mg} + \gamma$$

 $T = 2 \times 10^9 \,\mathrm{K}$, oxygen burning in core to give Si:

$$^{16}8O + ^{16}8O \rightarrow ^{28}14Si + ^{4}2He) \leftarrow \alpha$$
 particle reacts immediately & disappears

After O exhausted, $T \rightarrow 3 \times 10^9$ K, **photo-disintegration** of Si:

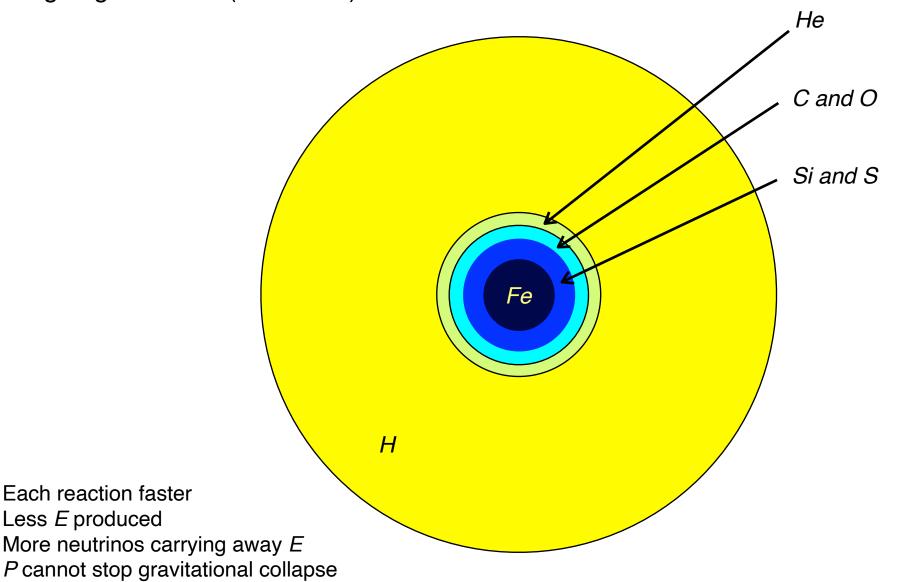
$$^{28}_{14}\text{Si} + \stackrel{?}{\gamma} \rightarrow ^{24}_{12}\text{Mg} + ^{4}_{2}\text{He}$$

 α particle reacts with Si and subsequent elements (silicon burning):

$$^{28}_{14}\text{Si} + ^{4}_{2}\text{He} \rightarrow ^{32}_{16}\text{S} + \gamma$$
 $^{32}_{16}\text{S} + ^{4}_{2}\text{He} \rightarrow ^{36}_{18}\text{Ar} + \gamma$
Fast sequence of fusion before **final explosion**
 $^{36}_{18}\text{Ar} + ^{4}_{2}\text{He} \rightarrow ^{40}_{20}\text{Ca} + \gamma$

Until $A \sim 56$ reached. Iron group elements: Fe, Cr, Mn, Co, Ni (when completed, $T = 7 \times 10^9$ K)

Highly evolved **supergiant** with different shells of heavier elements going to center (like onion) with iron core



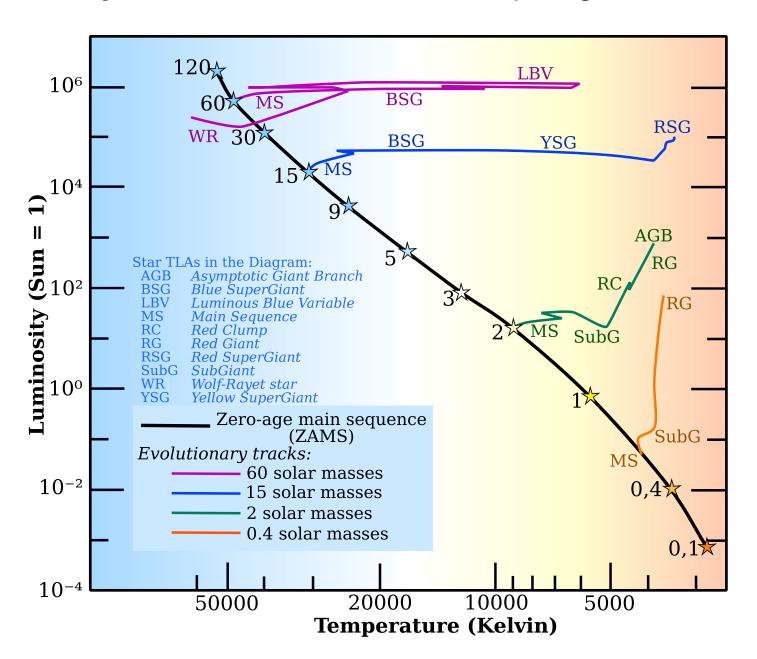
This condition will soon take to massive explosion & star destruction (supernova)

Life time of a star with mass $M = 25 M_{\odot}$ (before explosion as a supernova)

Chemical elements consumption time in the core

Hydrogen (main sequence).	6 million years
Helium	500,000 years
Carbon	600 years
Neon	1 year
Oxygen	6 months
Silicon	~ 1 day

Summary: evolution of stars in Hertzsprung-Russel diagram



Summary of element production in stars (before mass loss)

<i>M</i> < 0.5 M _☉	up to He
$0.5 \ M_{\odot} < M < 8 \ M_{\odot}$	up to about O
8 M _☉ < <i>M</i> < 10 M _☉	up to about Mg + small amount of heavier elements
<i>M</i> > 10 M _☉	wide range of elements, including Fe peak & heavier

Heaviest elements produced by different process: merger of stellar remnants after supernova (neutron stars) & nuclear reactions in new explosive event (*kilonova*)